

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 3

- Please attend the exercise class to which you have been assigned
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after 5pm Friday.
- These are more challenging problems of the kind found in this week's coursework.

(*)Problem 1: Obtain the limit as $n \rightarrow \infty$ for each of the following sequences:

$$(a) \quad a_n = \left(\frac{3n+1}{3n-1} \right)^n, \quad (b) \quad a_n = \frac{n^2}{2n-1} \sin \frac{1}{n}, \quad (c) \quad a_n = \frac{1}{n} \int_1^n \frac{1}{x} dx.$$

(*)Problem 2: Write out the first few terms of each of the following series and then find the sum:

$$(a) \quad \sum_{n=0}^{\infty} \left(\frac{5}{2^n} + \frac{1}{3^n} \right), \quad (b) \quad \sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}, \quad (c) \quad \sum_{n=1}^{\infty} \frac{40n}{(2n-1)^2(2n+1)^2}.$$

Problem 3: Obtain the limit as $n \rightarrow \infty$ for each of the following sequences:

$$(a) \quad a_n = \frac{\ln(n+1)}{\sqrt{n}}, \quad (b) \quad a_n = \frac{n!}{n^n},$$

$$(c) \quad a_n = n \ln \left(1 + \frac{7}{n} \right), \quad (d) \quad a_n = n - \sqrt{n^2 - n}.$$

Problem 4: Determine whether the following series converge (giving the sum) or diverge:

$$(a) \quad \sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}} \right)^n, \quad (b) \quad \sum_{n=0}^{\infty} \frac{n!}{1000^n}, \quad (c) \quad \sum_{n=1}^{\infty} \ln \left(\frac{n}{n+1} \right).$$

Problem 5: The figure below shows the first three rows and part of the fourth row of a sequence of rows of semicircles. There are 2^n semicircles in the n th row, each of radius $1/2^n$. Find the sum of the areas of all the semicircles.

